What you should have learned after Recitation 1

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Disclaimer

- The slides are written exclusively for 244 students. It might not be appropriate to use them in any earlier course.
- There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.

Logarithm Rules

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- **Read Section 10.5** on page 45 in the pdf file (page 733 in the book), **try all example problems**, and **do Exercise 44 - 61** on page 51 in the pdf file (Page 740 in the book).

Zero set of trigonometric function

- Zero set of $\cos x$:

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- Determine the set of points that blows $\tan(2x + \frac{\pi}{3})$ up.

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- **Read Section 4.3**, make sure you **memorize the table of the values of sine, cosine and tangent on usual special angles** on page 23 of the PDF file (page 279 in the book), and **do Exercise 17 - 26** on page 28 of the pdf file (page 284 in the book).

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- **Read Section 4.5**, make sure you can **recognize, distinguish different graphs of the trignometric functions and manipulate them by scaling and translation**, and do **Exercise 3 - 14, 23 - 16** on page 48 in the pdf file (page 304 in the book)

Solving polynomial equations by factorizations

Frankly speaking, factorization involves so many techniques that cannot be summarized in one single slide. So I'll skip that. Just do the following work

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First step: perform the long division, separate out a fraction whose nominator has the degree strictly less than the denominator.

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